DSP. Lec 10

* Analog filter design:-

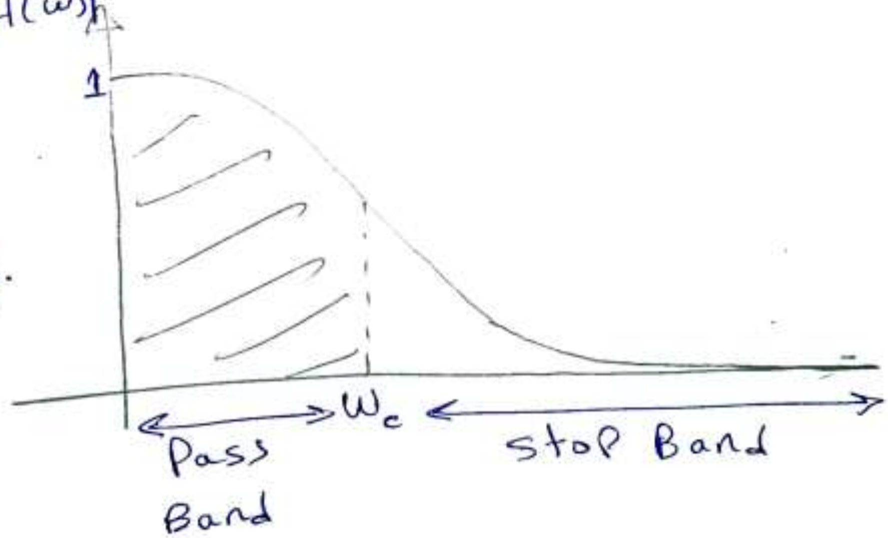
Digital filters are discrete time systems that make operations related to freq.

such as:

* low pass
* high pass
* Band pass
* Band stop

→ types of filters

[1] low pass filter (LPF)



gain = 1

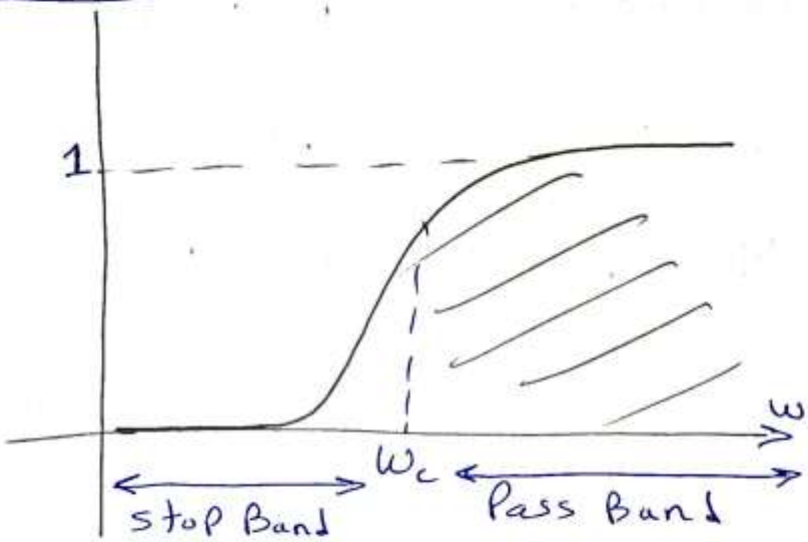
ω_c = Cut of freq.

↳ rad/sec

← يسمح بمرور الترددات ماقبل ω_c.

[2] High Pass Filter (HPF)

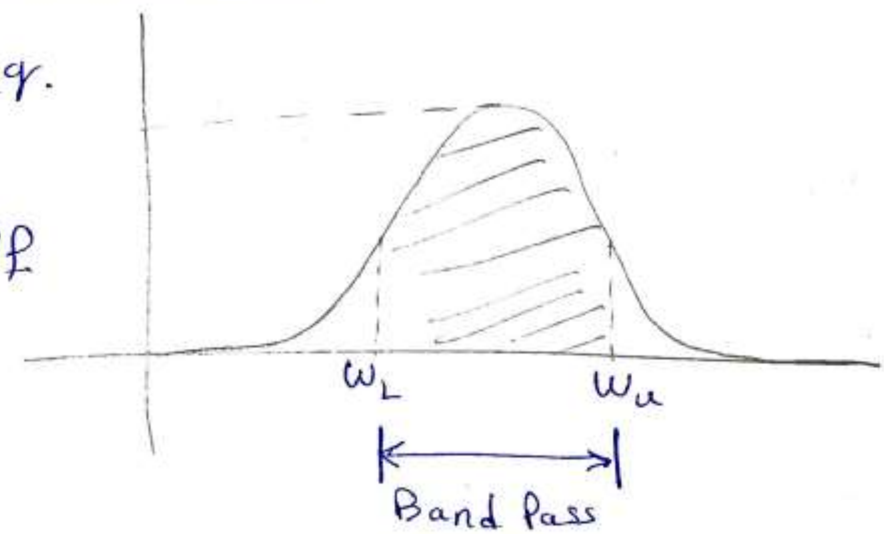
← يسمح بمرور الترددات فيما بعد W_c.



[3] Band Pass Filter (BPF):

$W_L \rightarrow$ lower cut of freq.
↳ rad/sec

$W_u \rightarrow$ upper cut off frequency.

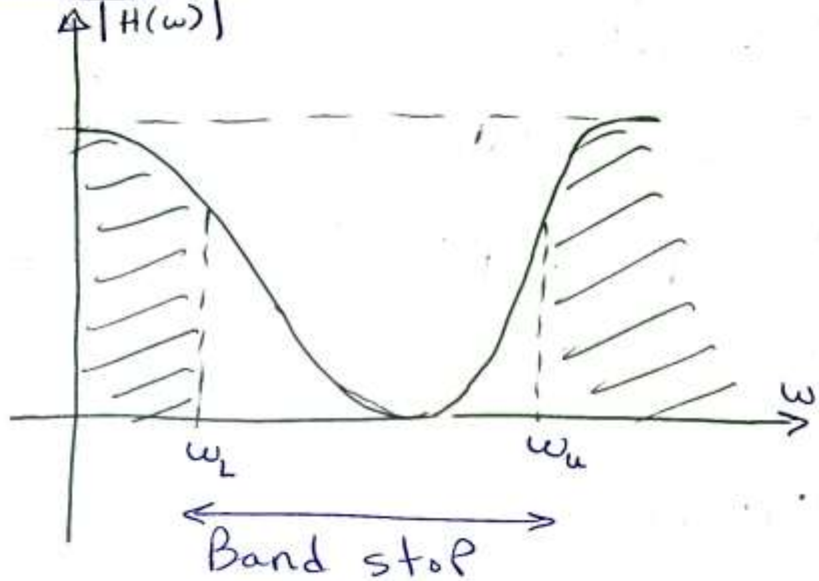


← يسمح بمرور الجزء ما بين W_L ، W_u.

[4]. Band stop filter (BSF)

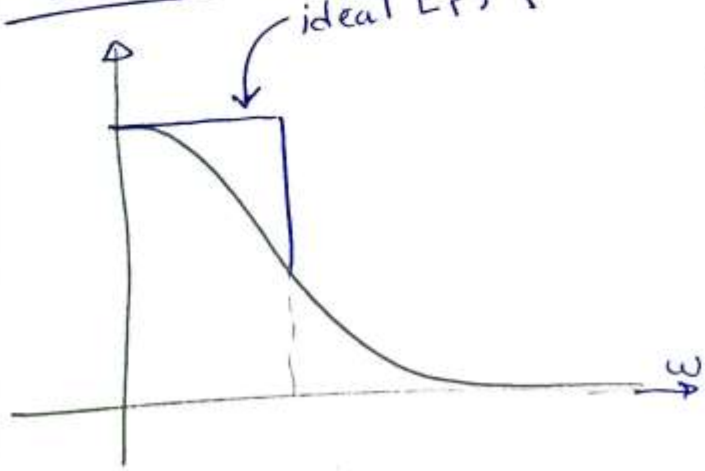
← يمرر فقط الجزء المخطط .

← يوقف الجزء من $W_L \to W_u$



[II] LPF

ideal LPF (ILPF)



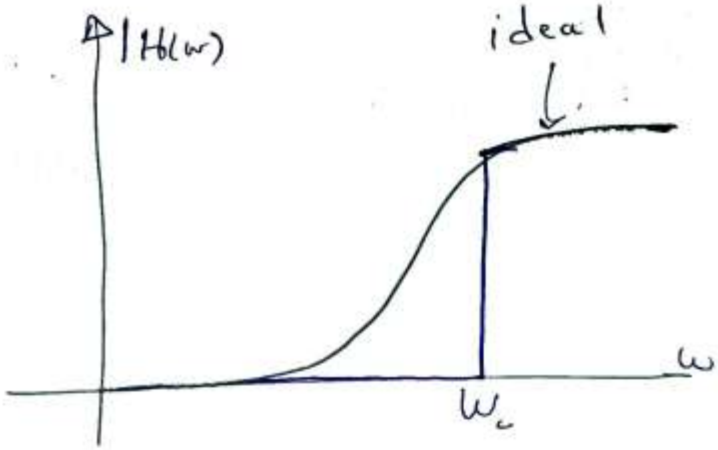
$H \Rightarrow$ T.F of filter.

→ In Design we create a system with A T.F that resembles the IDeal (Gain = 1)

→ Digital uses Approximation

* HPF

← ال (Accuracy) لا تكون
(100%) لذا نستخدم
ال (Approximation)



* Approximation methods for Design Analog filters

① Butter worth filter.

② Elliptic filter

③ Bessel filter

④ chebshev filter

* The specs. required for design

① Cut off freq. (W_c, W_L, W_u)

② order of the filter (Pole يحتوي على كام)

③ type of filter → LPF, HPF, BPF, BSF

[1] Butter worth filter design :-

The Design is to find T.F for the required filter, then we can implement it (Hardware or software)

* The design is made for LPF.

← هو مفترض انه يحدد نوع الـ (filter) في كل طريقة
لكنه بيشتغل على الـ (LPF) وفي الآخر لو طلع انه
مش (LPF) بيعمله (conversion) للنوع الناتج.

⇒ Design for LPF (using Butter worth filter)

The filter T.F :

$$H(s) = \frac{W_c^N}{(s-P_0)(s-P_1)\cdots(s-P_N)}$$

Where: N : the order of the filter

W_c : cut off frequency. (rad/sec)

$$P_K = W_c \, e^{J(N+2K+1)\frac{\pi}{2N}} \Rightarrow P \rightarrow P_0, P_1, \dots, P_N$$

$K = 0, 1, 2, \dots\dots, N-1$

$\boxed{EX}$ $W_c = 0.725$ rad/sec , $N = 2$

$$H(s) = \frac{(0.725)^2}{(s-P_0)(s-P_1)}$$

$\underline{\underline{K=0}}$

$$P_0 = (0.725)\, e^{J(2+0+1)\frac{\pi}{4}} = 0.725\left(\cos(135) + J\sin(135)\right)$$

$$\boxed{P_0 = -0.513 + J\,0.513}$$

$\underline{\underline{K=1}}$

$$P_1 = (0.725)\, e^{J(2+2+1)\frac{\pi}{4}} = -0.513 + J\,0.513$$

$\rightarrow 225$

$$H(s) = \frac{(0.725)^2}{(\underbrace{s+0.513}_{X} - \underbrace{J0.513}_{y})(\underbrace{s+0.513}_{X} + \underbrace{J0.513}_{y})}$$

$$H(s) = \frac{(0.725)^2}{(s+0.513)^2 + (0.513)^2}$$

$$\boxed{H(s) = \frac{0.526}{s^2 + 1.06s + 0.526}}$$

→ Normalized LPF ≡ NLPF

* is a low Pass filter with $\omega_c = 1$ rad/sec

<u>N = 1</u>

$$H(s) = \frac{1}{(s - P_0)}$$

$$P_0 = (1)\, e^{J(1+0+1)\frac{\pi}{2}} = e^{J\pi} = \cos(180) + \sin(180)$$

$$= -1$$

$$\boxed{H(s)\Big|_{NLPF \Rightarrow \omega_c = 1} = \frac{1}{s+1}}$$

← يبقى شكل ثابت .

$\underline{N=2}$

$$H(s)\Big|_{NLPF} = \frac{1}{(s-P_0)(s-P_1)}$$

$$P_0 = e^{J(2+0+1)\frac{\pi}{4}} \xrightarrow{} 135^\circ \quad = \frac{-1}{\sqrt{2}} + J\frac{1}{\sqrt{2}}$$

$$P_1 = e^{J(2+2+1)\frac{\pi}{4}} = \frac{-1}{\sqrt{2}} - J\frac{1}{\sqrt{2}}$$

$$H(s) = \frac{1}{\left(s+\frac{1}{\sqrt{2}} - J\frac{1}{\sqrt{2}}\right)\left(s+\frac{1}{\sqrt{2}} + J\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{1}{\left(s+\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}}$$

$$\boxed{H(s) = \frac{1}{s^2+\sqrt{2}\,s+1}}$$

NLPF (Butter worth)

$N = 1 \longrightarrow H(s) = \dfrac{1}{s+1}$

$N = 2 \longrightarrow H(s) = \dfrac{1}{s^2 + \sqrt{2}\, s + 1}$

$N = 3 \longrightarrow H(s) = \dfrac{1}{(s+1)(s^2+s+1)}$

مش هيزود عن $\boxed{N=3}$ في الإمتحان.

⇒ Convert from NLPF to any other type:-

NLPF

(1) LPF (with ω_c)

replace $s \longrightarrow \dfrac{s}{\omega_c}$

$$H(s)\Big|_{LPF} = H(s)\Big|_{NLP \; s \to \frac{s}{\omega_c}}$$

(2) HPF

$s \longrightarrow \dfrac{\omega_c}{s}$

$$H(s)\Big|_{HPF} = H(s)\Big|_{NLP \; s \to \frac{\omega_c}{s}}$$

NLPF

③ BPF

$$s \longrightarrow \frac{s^2 + \omega_u \omega_L}{s(\omega_u - \omega_L)}$$

$$H(s)\Big|_{BPF} = H(s)\Big|_{NLPF \atop s \to \frac{s^2 + \omega_u \omega_L}{s(\omega_u - \omega_L)}}$$

~~④ BSF~~

④ BSF

$$s \longrightarrow \frac{s(\omega_u - \omega_L)}{s^2 + \omega_u \omega_L}$$

$\boxed{EX}$ Design HPF with $\omega_c = 22$ rad/sec & $N = 3$

$$H(s)\Big|_{NLPF \atop N=3} = \frac{1}{(s+1)(s^2+s+1)}$$

$$H(s)\Big|_{HPF \atop \omega_c = 22 \text{ rad/sec}} = \frac{1}{\left(\frac{22}{s}+1\right)\left(\left(\frac{22}{s}\right)^2 + \frac{22}{s} + 1\right)}$$

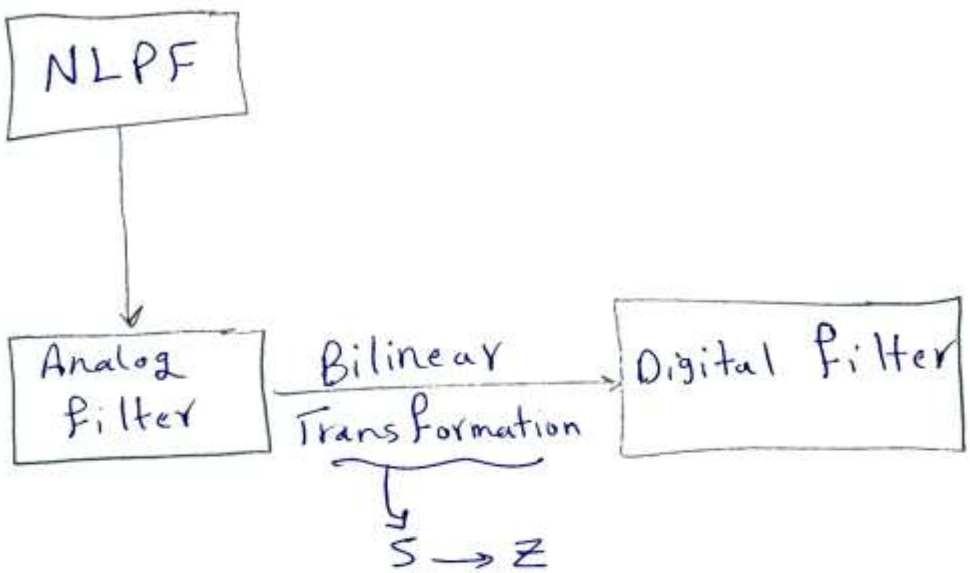
$$s \to \frac{22}{s}$$

بالضرب × s^3

$$H(s) = \frac{s^3}{(22+s)\left(22^2 + 22s + s^2\right)}$$

* Design Digital filters

→ The design is made in Analog Domain and then use biLinear transformation to Convert the design into Digital domain.



* BiLinear Transformation is a method That used to approximate the mapping from s-domain to Z-domain

$$S = \frac{2}{T}\left(\frac{Z-1}{Z+1}\right)$$

← (إثبات القانون) مش مفروض علينا لكنه للمعرفة .

$$Z = e^{Ts} = e^{\frac{T}{2}s}\, e^{\frac{T}{2}s}$$

$$= \frac{e^{\frac{T}{2}s}}{e^{\frac{-T}{2}s}}$$

<u>taylor</u> $e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

<u>assume</u> $x \leq 1 \longrightarrow \boxed{e^{x} = 1 + x}$

$$\therefore Z = \frac{e^{\frac{T}{2}s}}{e^{\frac{-T}{2}s}} = \frac{1 + \frac{T}{2}s}{1 - \frac{T}{2}s}$$

$$Z - \frac{T}{2}sZ = 1 + \frac{T}{2}s$$

$$Z - 1 = \frac{T}{2}s\left(Z + 1\right)$$

$$\boxed{s = \frac{2}{T}\,\frac{(Z-1)}{(Z+1)}}$$

Given

Design specs are in digital domain

ω_{c_D} , ω_{u_D} , ω_{L_D} , N, type, T

$\underbrace{\omega_{c_D}}_{\text{Digital domain}}$

$\omega_c, \omega_u, \omega_L \longrightarrow$ in Analog (عشان متتلخبطش)

$$W_A = \frac{2}{T} \tan\left(\frac{W_D T}{2}\right)$$ $\rightarrow$ to convert between analog and digital frequency

$T \Rightarrow$ Sampling Time

$\boxed{Ex}$ Design digital Band Pass filter with $N = 1$ with following specs:-

- $f_{L_D} = 2.4$ KHz (lower cut of freq.)
- $f_{u_D} = 2.6$ KHz (upper cut of freq.)
- $f_s = 8$ KHz (sampling frequency)

$\hookrightarrow T = \frac{1}{f_s}$

$\boxed{1}$ Convert $\underbrace{\omega_{L_D} \;\&\; \omega_{u_D}}_{\text{Digital domain}} \Rightarrow \underbrace{\omega_L, \omega_u}_{\text{Analog domain}}$

$$\cancel{\omega_L =}\ T = \frac{1}{8 * 10^3} \text{ sec}$$

$$\cancel{\omega_L = \frac{2}{\frac{1}{8 * 10^3}} \tan\left(\frac{2.4 * 10^3}{8 * 10^3 * 2}\right)} =$$

$$\omega_L = \frac{2}{\frac{1}{8 * 10^3}} \tan\left(\frac{2\pi * 2.4 * 10^3 * \frac{180}{\pi}}{8 * 10^3 * 2}\right)$$

$$= 22022.1107 \text{ rad/sec}$$

$$\omega_u = \frac{1}{\frac{1}{8 * 10^3}} \tan\left(\frac{2\pi * 2.6 * 10^3}{8 * 10^3 * 2} * \frac{180}{\pi}\right)$$

$$\omega_u = 26109.63 \text{ rad/sec}$$

[2] NLPF with $N=1$

$$H(s) = \frac{1}{s+1}$$

$$H(s)\Big|_{BPF} = H(s)\Big|_{\substack{NLPF \\ s \longrightarrow \frac{s^2+\omega_u\omega_L}{s(\omega_u-\omega_L)}}}$$

$$H(s) = \frac{(\omega_u-\omega_L)\,s}{s^2+(\omega_u-\omega_L)\,s+\omega_u\omega_L}$$

$$H(s)\Big|_{ABPF} = \frac{4087.516\,s}{s^2+4087.516\,s+57498909 6.1}$$

Analog

[3] using B.T (Bilinear transformation)

$$s \longrightarrow \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

$$H(z)\Big|_{DBPF} = H(s)\Big|_{ABPF \atop s \to \frac{2}{T}\left(\frac{z-1}{z+1}\right)}$$

$$\boxed{H(z) = \frac{0.0728 z^2 - 0.0728}{z^2 + 0.7118 z + 0.854}}$$

Ex Design first order digital LPF with $\omega_c = 30\pi$ rad/sec and sampling time $T = \frac{1}{90}$ sec.

$$\boxed{\begin{array}{l} \omega_{c_D} = 30\pi \text{ rad/sec} \\ T = \frac{1}{90} \text{ sec} \end{array}}$$ "given"

① obtain ω_c:

$$\omega_c = \frac{2}{T} \tan\left(\frac{\omega_{c_D} T}{2}\right)$$

$$\omega_c = 180 \tan\left(\frac{30\pi}{90*2} * \frac{180}{\pi}\right) \simeq 103.923 \text{ rad/sec}$$

$\boxed{2}$ N=1 , NLPF

$$H(s)\Big|_{\substack{NLPF \\ \text{with } N=1}} = \frac{1}{s+1}$$

$$H(s)\Big|_{\substack{LPF \\ \text{with } \omega_c = 103.923}} = H(s)\Big|_{\substack{NLPF \\ s \to \frac{s}{\omega_c}}}$$

$$= \frac{1}{\frac{s}{\omega_c}+1} = \frac{\omega_c}{s+\omega_c}$$

$$\boxed{H(s)\Big|_{LPF} = \frac{103.923}{s+103.923}}$$

$\boxed{3}$ using Bilinear transformation

$$H(z) = H(s)\Big|_{\substack{LPF \\ s \to \frac{2}{T}\left(\frac{z-1}{z+1}\right)}}$$

$$H(z) = \frac{103.923}{180\left(\frac{z-1}{z+1}\right) + 103.923}$$

$$\boxed{H(z)\Big|_{DLPF} = \frac{0.366\,(z+1)}{z - 0.2679}}$$